

On a Source of Systematic Error in Absolute Measurement of Galactocentric Distance from Solving for the Stellar Orbit Around Sgr A*

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Abstract. Eisenhauer et al. (2003, 2005) derived absolute (geometrical) estimates of the distance to the center of the Galaxy, R_0 , from the star S2 orbit around Sgr A* on the assumption that the intrinsic velocity of Sgr A* is negligible. This assumption produces the source of systematic error in R_0 value owing to a probable motion of Sgr A* relative to the accepted velocity reference system which is arbitrary to some extent. Eisenhauer et al. justify neglecting all three spatial velocity components of Sgr A* mainly by low limits of Sgr A*'s proper motion of 20–60 km/s. In this brief paper, a simple analysis in the context of the Keplerian dynamics was used to demonstrate that neglect of even low (perhaps, formal) radial velocity of Sgr A* leads to a substantial systematic error in R_0 : the same limits of 20–60 km/s result in R_0 errors of 1.3–5.6%, i.e., $(0.1\text{--}0.45) \times (R_0/8)$ kpc, for current S2 velocities. Similar values for Sgr A*'s tangential motion can multiply this systematic error in the case of S2 orbit by factor $\approx 1.5\text{--}1.9$ in the limiting cases.

1. Introduction

The distance from the Sun to the center of the Milky Way, R_0 , is a fundamental Galactic constant for solving many astronomical and astrophysical problems (see, e.g., Reid 1993). That is why, in its turn, the problem of determination of R_0 remains topical over many years. Absolute (i.e., not using luminosity calibrations) estimates of R_0 with a current 3% formal uncertainty from modelling the star S2 orbit around the compact concentration of dark mass, the so-called “supermassive black hole”, associated with the radio source Sgr A* (Eisenhauer et al. 2003, 2005; Trippe et al. 2006) present a major breakthrough in measuring R_0 ! (For brevity, from here on the object in focus of S2 orbit will be referred to as “Sgr A*”.)

However, even though to take no notice the issue on coincidence of Sgr A* with the dynamical and/or luminous center(s) of our Galaxy (see discussion in Nishiyama et al. 2006), taken alone the modelling the orbital motion of a star near Sgr A* can be plagued with various *systematic* sources of error. Since Eisenhauer et al. solved for the *Keplerian* orbit of the star S2, in the literature *relativistic* effects and *non-Keplerian* orbit modelling are primarily explored for this problem (e.g., Eisenhauer et al. 2005; Mouawad et al. 2003; Weinberg et al. 2005).

Meanwhile, Eisenhauer et al. also used another *assumption that the intrinsic velocity of Sgr A* is negligible*. This assumption can produce the source of systematic error in R_0 value owing to a probable motion of Sgr A* relative to the accepted velocity reference system which is arbitrary to some extent. Thus far, no consideration has been given to the role of this factor in measuring R_0 .

In this study, a simple analysis is used to evaluate the *impact of an unaccounted motion of Sgr A** (i.e., the focus of S2 orbit) *on an R_0 value* found from the formal solution of orbit. The Keplerian dynamics only is taken into consideration because relativistic and non-Keplerian effects seem to be insignificant for measuring R_0 (Eisenhauer et al. 2005; Mouawad et al. 2003; Weinberg et al. 2005). Particular attention has been given to the impact of a nonzero radial velocity of Sgr A* relative to the Local Standard of Rest.

2. Structure of the Problem on Determination of Orbital Parameters, Distance to and Mass at Orbital Focus (Sgr A*)

The completeness of solution of the problem in question is determined by the type of available data on motion of an individual star (S2).

2.1. Star's Proper Motions Alone are Available

In this case, *all six orbital parameters are solved, except that only the absolute value of the inclination angle, i , is determined*, leaving the questions of the direction of revolution (prograde, $i > 0$, or retrograde, $i < 0$) and where along the line of sight the star is located behind the central object unresolved (e.g., Ghez et al. 2003). Besides, *the semimajor axis is derived in angular units* (in arcsec), hereafter a'' . The distance to the focus, i.e., R_0 , and the central mass, M , can not be solved.

With accepted R_0 , however, the value of semimajor axis, a , is calculated in linear units (in kpc) and the central mass is found from Kepler's third law

$$M = n^2 a^3 / G, \quad n = 2\pi / P, \quad (1)$$

where G is the gravitational constant, n is the mean motion, and P is the orbital period, as it has been done in Schödel et al. (2002).

2.2. Proper Motions and at Least a Single Measurement of Radial Velocity of Star are Available

In this case, *the problem is completely solved* if the value of star's radial (line-of-sight) velocity, V_r , is significantly different from zero (more exactly, from the radial velocity of the focus).

A. *The sign of V_r determines the sign of i .* Consequently, this also breaks the ambiguity in the direction of rotation and in star's location along the line of sight relative to the focus (e.g., Ghez et al. 2003).

B. *The absolute value of V_r determines values R_0 and M .* To gain greater insight into the fact of the matter, the problem can be symbolically divided into two subproblems: (1) the determination of orbital parameters from the proper motions alone and (2) the determination, knowing the orbit, of the distance to focus (R_0) and of the central mass from the measurement(s) of V_r . These subproblems are almost independent in the case of modelling the motion of stars around Sgr A*, since up to now proper motion measurements are numerous, but V_r ones are few or at all V_r actually is single, for any S star with solved orbit. So, V_r measurement(s) contribute(s) almost nothing to the knowledge of orbit, and vice versa proper motion measurements do not directly determine neither R_0 nor M . Thus, such breaking the problem down seems to be quite realistic.

If so, the value of $|V_r|$ may be considered as determining R_0 and M from known orbital parameters as follows.

(i) The orbit elements enable to find the ratio between $|V_r|$ and the total space velocity, V , for the moment t :

$$V_r^2/V^2 = \frac{[e \sin v \sin u + (1 + e \cos v) \cos u]^2 \sin^2 i}{1 + 2e \cos v + e^2}, \quad (2)$$

where e is the eccentricity, v is the true anomaly, $u = v + \omega$ is the argument of latitude, ω is the argument of pericenter. A value of v can be calculated from classical formalism:

$$\tan(v/2) = \sqrt{(1+e)/(1-e)} \tan(E/2),$$

$$E - e \sin E = \mathcal{M}, \quad \mathcal{M} = n(t - t_0) + \mathcal{M}_0,$$

where E and \mathcal{M} are the eccentric and mean anomalies, correspondingly (e.g., Subbotin 1968). Consequently, the knowledge of $|V_r|$ determines V .

(ii) The value of total velocity V can be expressed as

$$V = na \left(\frac{1 + 2e \cos v + e^2}{1 - e^2} \right)^{1/2}. \quad (3)$$

From this equation, the value of a in linear units can be calculated. Then the ratio between a values in linear and angular units gives R_0 :

$$R_0 = \frac{a \text{ [kpc]}}{a''}. \quad (4)$$

(iii) Using Eq. (1) with a in linear units determines the central mass M .

3. Systematic Error in R_0 Owing to a Nonzero Motion of Orbital Focus (Sgr A*)

3.1. Nonzero Radial Velocity of Sgr A*

Eisenhauer et al. (2003, 2005) assume that the radial velocity of Sgr A*, $V_r^* \equiv V_r(\text{Sgr A}^*)$, relative to the Local Standard of Rest (LSR) is zero. Neglect of a possible radial motion of Sgr A* is equivalent to the introducing a corresponding systematic error in all V_r values. This error is equal to a value of V_r^* and is the same in all measurements of V_r . From Eqs. (2)–(4) follows that the relative systematic error in V_r velocity fully converts to the relative systematic error in R_0 , i.e.,

$$\delta_{\text{sys}} \equiv \frac{\sigma_{\text{sys}}(V_r)}{|V_r|} = \frac{\sigma_{\text{sys}}(R_0)}{R_0}. \quad (5)$$

These simple considerations make it possible readily to evaluate the systematic error in R_0 knowing typical values of V_r used for the determination of distance to S2/Sgr A*. The first S2 radial velocity measurement of $V_r = -510 \pm 40$ km/s by Ghez et al. (2003) was obtained just 30 days after the star's passage through the pericenter point when V_r was changing very rapidly. Therefore, this measurement contributes to the solution for R_0 much less than subsequent ones, hence the evaluation of $\sigma_{\text{sys}}(R_0)$ must lean upon these latter. Besides, the subsequent radial velocities, having substantially higher absolute values, give a lower limit for $\sigma_{\text{sys}}(R_0)$.

Eisenhauer et al. justify neglecting all three spatial velocity components of Sgr A* mainly by low limits of Sgr A*'s proper motion of 20–60 km/s (Eisenhauer et al. 2005).

Table 1. Systematic error in R_0 because of neglect of a possible radial motion of Sgr A*

Observational Period	$\langle V_r \rangle$ (km/s)	$V_r(\text{Sgr A}^*)$ (km/s)	δ_{sys}	$\sigma_{\text{sys}}(R_0)$ (kpc)	
				$R_0 = 7.5$ kpc	$R_0 = 8$ kpc
2003 April–June	−1500	20	0.013	0.10	0.11
		60	0.040	0.30	0.32
2004 July–August	−1075	20	0.019	0.14	0.15
		60	0.056	0.42	0.45

Such values of radial velocities seem to be quite plausible for massive objects in the Galactic center (see Blitz 1994). Table 1 presents values of systematic errors in R_0 calculated for possible Sgr A*’s radial velocities of $V_r^* = 20$ and 60 km/s with $R_0 = 7.5$ and 8.0 kpc (Reid 1993; Nikiforov 2004; Trippe et al. 2006). In Table 1, $\langle V_r \rangle$ is the average of velocities V_r , used for estimation of R_0 in Eisenhauer et al. (2005), over the observational period.

Table 1 demonstrates that *neglect of even moderately low radial velocity of the orbital focus (Sgr A*) relative to the LSR can lead to a substantial systematic error in R_0* : values of $V_r^* = 20$ –60 km/s result in systematic R_0 errors of 1.3–5.6%, i.e., **(0.1–0.45) \times ($R_0/8$) kpc**, for current typical star’s velocities. Notice that the value of $\sigma_{\text{sys}}(R_0)$ can not be reduced statistically since *all* V_r values is biased coherently by any nonzero velocity of Sgr A*. Only solving for $V_r(\text{Sgr A}^*)$ can correct this systematic error in R_0 !

It should be mentioned that Trippe et al. (2006) state that they already solved 3D velocity of Sgr A*, however, not presenting in their short paper any details—no values of velocities and even no exact value of current point estimate for R_0 !

3.2. Nonzero Proper Motion of Sgr A*

The reference frame for proper motions Eisenhauer et al. have established by measuring the positions of nine astrometric reference stars relative to typically 50–200 stars of the stellar cluster surrounding Sgr A*; the uncertainty of the reference frame is 11.7 km/s (see Eisenhauer et al. 2003). The effect of nonzero proper motion Sgr A* relative to this frame, $\vec{\mu}^* \equiv \vec{\mu}(\text{Sgr A}^*)$, can be approximately estimated if to imagine that the value of R_0 is determined, also on the basis of V_r ’s measurement at a moment t , not from Eqs. (3) and (4) but from the ratio between star’s linear velocity on the sky, V_μ , and star’s proper motion, μ , measured for the same moment t :

$$R_0 = \frac{V_\mu}{\mu}. \quad (6)$$

The value of V_μ is a known function of V_r , orbital elements, and time:

$$V_\mu^2 = V^2 - V_r^2 = V_r^2(\Psi^{-2} - 1), \quad \Psi^2(t) \equiv \frac{V_r^2}{V^2}, \quad (7)$$

where $\Psi^2(t)$ can be calculated from orbital elements [Eq. (2)]. Any nonzero radial velocity V_r^* and nonzero proper motion μ^* of Sgr A* are equivalent to the introducing systematic errors ε_{V_μ} and ε_μ in V_μ and μ , correspondingly. Because values of V_r^* and μ^* are independent and unknown, their combined impact on an R_0 estimate can be described by the formula of

propagation of errors applied to Eq. (6):

$$\begin{aligned}\varepsilon_{R_0}^2 \equiv \sigma_{\text{sys}}^2(R_0) &= \left(\frac{\varepsilon_{V_\mu}}{\mu}\right)^2 + \left(\frac{V_\mu}{\mu^2}\varepsilon_\mu\right)^2 \\ &= (R_0/V_\mu)^2(\varepsilon_{V_\mu}^2 + R_0^2\varepsilon_\mu^2).\end{aligned}\quad (8)$$

From Eq. (7) follows

$$\varepsilon_{V_\mu} = \varepsilon_{V_r} \sqrt{\Psi^{-2} - 1}, \quad (9)$$

if an uncertainty on orbit elements is ignored, as it was actually done in section 3.1. Then considering that $\varepsilon_{V_r} = |V_r^*|$ we have

$$\varepsilon_{R_0}^2 = \frac{R_0^2}{V_r^2} \left(V_r^{*2} + R_0^2 \varepsilon_\mu^2 \frac{\Psi^2}{1 - \Psi^2} \right). \quad (10)$$

Value of ε_μ depends from the relative orientation of vectors $\vec{\mu}$ and $\vec{\mu}^*$. In the general case $0 \leq \varepsilon_\mu \leq \mu^*$. Hence, e.g., for equal radial and tangential components of Sgr A* motion, i.e., for $V_\mu^* = |V_r^*|$, or $\mu^* = |V_r^*|/R_0$,

$$\max \varepsilon_{R_0} = \varepsilon_{R_0}(V_r^*) k_1, \quad k_1 = \frac{1}{\sqrt{1 - \Psi^2}}, \quad (11)$$

$$\varepsilon_{R_0}(V_r^*) \equiv R_0 \left| \frac{V_r^*}{V_r} \right|. \quad (12)$$

Here $\varepsilon_{R_0}(V_r^*)$ is the systematic error in R_0 owing to only the radial velocity of Sgr A* [see Eq. (5)].

For $V_\mu^{*2} = 2V_r^{*2}$, or $\mu^* = \sqrt{2}|V_r^*|/R_0$, i.e., for equal all three Cartesian components of Sgr A* motion,

$$\max \varepsilon_{R_0} = \varepsilon_{R_0}(V_r^*) k_2, \quad k_2 = \sqrt{\frac{1 + \Psi^2}{1 - \Psi^2}}. \quad (13)$$

With the S2 orbit elements derived in Eisenhauer et al. (2005), $k_1 \approx 1.4974$, $k_2 \approx 1.8666$.

Thus, for a given V_r the effect of nonzero proper motion of Sgr A* on R_0 , being a function of the true anomaly, ranges from zero to values comparable to the effect of nonzero radial velocity of Sgr A*, in the latter case increasing measurably the total systematic error in R_0 .

4. Conclusions

Simple considerations show that *neglect of even low radial velocity of Sgr A* relative to the LSR leads to a substantial systematic error in R_0* —up to 6%, i.e., ~ 0.5 kpc, for plausible values of Sgr A* velocity. It is too much to consider the distance to Sgr A*, not to mention the value of R_0 , as being established reliable from the present results on modelling the S2/Sgr A* system.

A proper motion of Sgr A* biases the distance value not so inevitably, but in limiting cases can increase the systematic error in R_0 owing to radial motion by factor up to ≈ 1.5 – 1.9 for similar values of Sgr A*'s tangential velocity.

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